Proc. SPIE, vol. 1987, 14-25, 1993

Characterization of a Large Format CCD Array

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ABSTRACT

A study has been made of some of the factors affecting the quality of the recorded image when using a large format 2K x 2K Kodak CCD imager. The optimal means for removing the DC offset, often referred to as the bias, is to sample many such frames and take a median at each pixel. The effects of external noise from electronics and cosmic rays are then removed well. The remaining noise in the frame appears to have a gaussian form. The effect of pixels that accumulate signal due to a high thermal current can be significantly reduced by cooling. To remove the non-uniformity across the array due to the variation in responsivity from pixel to pixel it is extremely important to acquire images of uniform fields at the wavelength at which the observations of interest were made. This is especially important when image acquisition is over wavelength ranges outside the normal visual. For highly accurate photometric applications, residual image effects needed to be considered.

1. INTRODUCTION

To pursue research interests in the areas of medical digital radiography and astronomical imaging a Photometrics cooled CCD system was acquired approximately one year ago. The camera head is a model CH-250 and is equipped with a Kodak KF-4200 CCD. This chip is built using two phase, two polysilicon NMOS technology, and has 2033 (H) x 2044 (V) usable pixels each of size 9 μm square. This front illuminated device has a quantum efficiency greater than 20% between 500 nm and 920 nm. In this camera the chip is operated cooled using a three stage thermoelectric cooler and a heat sink consisting of a mix of ethyl-glycol and water, circulated using an external pump. The temperature of the chip can either be regulated to operate at -35°C, or with the temperature controller disabled, and on a cold night at the observatory, can fall below -60°C. The gain settings are such that 1 AD unit (1 output count) is the equivalent of 5 electrons of charge on the output of the CCD.

In order to understand to what extent this device might be useful in various scientific imaging projects, some time has initially been spent characterizing the CCD performance. One of the attractions of this system was the low dark current anticipated without the need for cryogenic cooling. This feature would be appealing for proposed applications in a mobile digital radiography unit in the hospital or as an imager at a remote, robotic observatory where cryogen would be difficult to maintain.

This paper will report on the read noise performance of the KF-4200 chip, the dark current as a function of temperature and the presence of ghost images under certain illumination conditions.

2. DATA REDUCTION

Images collected by the CCD need to be corrected for a number of instrumental effects before analysis of the information content can proceed. As is true with most CCDs charge is transferred from each individual pixel onto the output node (floating diffusion) where an on-chip amplifier buffers it to the outside world. Certain sources of noise associated with the output circuitry can be minimized through the use of the correlated double sampling algorithm[1] (CDS). The fundamental noise source is then determined by the on-chip amplifier's white and 1/f noise contribution, and the CDS sampling function. For any particular CCD, this noise is a function of device operating temperature and the rate at which pixels are readout[2]. The noise is minimized by cooling the chip and using a read-out rate out as slow as permitted by the application.

As is the case with most CCD systems, the output signal from the KF-4200 chip is biased slightly positive so that the read-out noise associated with the on-chip output amplifier never drives the A/D input into the negative. Normally an exposure of short duration with no incident light is acquired of the entire frame which is used to correct for the variation in this DC level from pixel to pixel. This is often referred to as the bias frame. As the KF-4200 is a full frame CCD, such frames are acquired by having a shutter in front of the CCD closed and performing a read-out at the maximum rate permitted by the A/D (500 kHz in the camera used here).

In addition to the bias level at each pixel, a signal accumulates in each pixel value that is attributable to the generation of electrons due to thermal phonons in the silicon. This signal, which is present with no light incident on the array, is referred to as the dark signal, and varies considerably across the array. Each pixel has a characteristic dark current which is a strong function of temperature. Normally dark frames are gathered to allow for subtraction of this signal from the data. Dark signal is undesirable as it both contributes an additional noise source, and acts to fill the well of the affected pixel thereby reducing the maximum possible dynamic range.

Finally each pixel has a slightly different responsivity to the incident radiation. These variations are attributable to physical differences between pixels as result of fabrication, dust particles on the surface of the CCD and on the protective vacuum window in front of the CCD, and gradients in thickness of the overlying absorbing oxide. Normally an image of a known uniformly radiating field is obtained ; the resulting difference in measured pixel intensity is then attributed to the pixels themselves.

When analyzing long exposures the optimal data reduction expression to obtain calibrated images with the array used here is :

$$x_{ij}(\lambda) = \frac{x'_{ij}(\lambda,t,T) - \overline{dark_{ij}(t,T)}}{flat_{ij}(\lambda,T) - \overline{bias_{ij}(T)}} \tag{1}$$

where x'_{ij} and x_{ij} are the original and calibrated signal at each pixel., *ij*. The argument t indicates the integration time of each frame, λ is the passband through which the image was obtained and T is the temperature of the CCD imager. *Flat* corrects for pixel sensitivity variations, *dark* removes thermal effects and *bias* eliminates DC offset effects. Thus x_{ij} represents the image corrected for pixel dark current, bias and sensitivity variations.

For each CCD pixel read-out there is a random noise term added attributable to the noise spectrum of the on-chip amplifier. When the accumulated photo-signal in the pixel is large this additional noise contribution is small compared to the inherent uncertainty due to the Poisson distribution of signal photons. In most scientific applications the dynamic range however is critical and it is therefore imperative to detect the smallest signal (fewest photons) possible. The above expression for data analysis indicates that suitable bias and dark frames need to be subtracted in the process of reduction. It is important that these DC offset corrections do not affect the ultimate S/N in the frame by contributing additional noise to the final image.

As the source of read noise is random, it is possible to acquire a number of bias and dark frames, and combine them to produce a final frame that has minimized the read noise contribution. Unfortunately there are other sources of noise that make this procedure difficult[4]. The incidence of high energy cosmic rays onto the upper atmosphere produce a flux of secondaries (75% of which are muons) at the earth's surface. The muons have a mean energy at sea level of 2 GeV and a total flux of 1.8 x 10^{-2} cm^{-2} sec^{-1}. They interact fairly strongly with silicon[3] with a range of $\approx$ 1.2 gcm^{-2}. Their passage through the silicon produces conduction band electrons that are collected, along with photon produced electrons, and result in the final image having bright pixels corresponding to cosmic ray interaction. Many events affect only a single pixel, but some appear streaked as a result of non vertically incident muons. The muon's angular distribution at sea level has a $\cos^2\theta$ form. For an 18 mm x 18 mm CCD these figures imply a event rate on a frame of $\approx$ 100 events cm^{-2} $hour^{-1}$. This rate is dependent on the altitude above sea level at which the detector is located and, to a weaker extent, its latitude on the earth.

In addition to cosmic rays a variety of electrical sources of noise, if present during the read-out and digitization of the CCD output, can corrupt the data. An example of such a source is the motors that turn a dome at an astronomical observatory. With our CCD system rotating the dome while performing a read-out corrupts (makes them appear bright) large numbers of pixels in certain rows. Ideally after sufficient experience with the detector and field site where the data is being gathered these electrical interference effects can be minimized.

The effect of cosmic rays is more difficult to eliminate. This is particularly true in the case of obtaining noise free dark frames. When exposure times are long the number of cosmic ray events recorded by the CCD becomes large. In addition if multiple dark frames are gathered to reduce the read noise level, each frame will have associated with it randomly occurring cosmic ray events. If a straight mean is used on the multiple frames to reduce the read noise, the mean for many of these pixels will be corrupted by one, or more, cosmic ray hits. An alternate approach is to median filter the multiple frames[4].

3. MEDIAN FILTERING

The noise reduction capabilities of the mean and median[5,6] can be understood by considering the dark current removal task as a statistical estimation problem. Given a stochastic process, for example a signal corrupted by noise, we often want to estimate the underlying signal. If the noise can be assumed to have a zero mean value, then the underlying signal value is the mean value of the noise signal. To recover our signal we need to estimate the mean value of the noise signal. The usual way to do this is to record several independent data (realizations of the stochastic process) and calculate the sample mean:

$$\bar{x} = \sum_{i=1}^{N} x_i$$

If the stochastic process $\{x\}$ is stationary over the interval of observation and the process follows a normal probability function, then $\bar{x}$ will approach the true mean μ_x, as N increases. Since x is a random variable, $\bar{x}$ is also a random variable with an associated $\sigma_{\bar{x}}^2$. The well known relationship between the variance of x, σ_x^2, and the variance of the estimate of the mean is :

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{N}$$

If we know (or can assume) σ_x^2 then we can determine the number of independent samples required to estimate $\bar{x}$ with a given precision, i.e.

$$N = \frac{\sigma_x^2}{\sigma_{\bar{x}}^2}$$

The above mean estimate depends on a stationary stochastic distribution. In our case the presence of additional detected cosmic events will significantly change the calculated sample mean values. Since the cosmic events are not part of the dark current process, their presence will cause a local positive bias in the estimated mean dark current. This will be subsequently subtracted from each normally exposed image level at that pixel. The net effect will be increased fluctuations in the dark frames.

This situation can be alleviated to some extent by using a different estimate for the underlying dark current ; the median. The median is a statistic whose utility lies in the fact that it is less corrupted by a few outlying data values[5] (cosmic ray events here). For an odd number of data values the median is the central value when all are arranged in ascending or descending order. If we are to use a median, we can evaluate its performance by comparing it to the sample mean.

As the number of data points N increases, the median approaches the mean value. In the absence of outliers, the error of the median, as an estimate of the mean, is greater than that of the sample mean. Table 1 lists the standard deviation associated with the mean and median for a process with underlying variance, $\sigma_x^2 = 1$.

Table 1: RMS error associated with the sample mean and median of a variable with unity variance, $\sigma_x^2=1$, for several sample sizes.

N	$\sigma_{\bar{x}} = \frac{1}{\sqrt{N}}$	σ_{median}	$\frac{\sigma^2_{median}}{\sigma^2_{\bar{x}}}$
3	0.577	0.670	1.35
5	0.447	0.536	1.44
7	0.378	0.459	1.47
9	0.333	0.408	1.50
11	0.302	0.374	1.53
13	0.277	0.348	1.57

For a large number of samples the standard deviation of the median is 1.253 times that of the sample mean. This implies that for an equivalent error the median would require $(1.253)^2$N or 1.57N samples, where here N is the number of samples used to calculate the sample mean. For example if N=3, then from the last column of table 1, 1.35 x 3 = 4 samples would be required for the median of equivalent error.

The above characteristics for the sample mean and median can be interpreted in terms of the residual fluctuations formed by the original noisy signal minus the estimate (sample mean or median). This calculation is useful in validating the results using a practical imager.

Consider a signal x_{ij}, corrupted by random dark current fluctuations :

$$x_{ij} = d_{ij} + \varepsilon_{ij}$$

where ε_{ij} is a zero mean random variable. A sample mean or median can be calculated for each pixel, as an estimate of d_{ij}. To experimentally verify the magnitude of x_{median} for several N we need many realizations of x_{median} . If we want to use the set of median values for each pixel across the image, the pixel-to-pixel bias, d_{ij}, needs to be eliminated. If we form the residual for each pixel,

$$y_{ij} = x_{ij} - x_{median,ij}$$

then this value will not contain the pixel-to-pixel bias. The corresponding value for the residual of the mean is :

$$y_{ij} = x_{ij} - \overline{x_{ij}}$$

It can be shown that the ratio of the variance values for the median and mean residuals for N large is :

$$\frac{\sigma^2_{y,mean}}{\sigma^2_{y,median}} = \frac{N}{N+0.253} \tag{2}$$

The above expression can be compared with equivalent experimental data collected with the Kodak imager. Two sets of suitable data were available. One set consisted of seven bias frames taken with the CCD at -35°C, and the other, of thirteen bias frames taken with the CCD at -60°C. In order to eliminate any edge effects only the central 1000 x 1000 pixel region of the frames are further considered. Figure 1(a) shows a histogram of these central pixels for a single bias frame, and the histogram of the mean and median values for each pixel in the seven frame data set at -35°C. Figure 1(b) is similar to figure 1(a) except that it is for a single frame, and the thirteen frame set taken at -60°C.

Figure 1 : Figure 1(a) is a histogram of the central 1000 x 1000 pixels of each of, a single bias frame and the mean and median for each pixel of 7 bias frames taken with the CCD cooled to -35°C. Figure 1(b) is similar to figure 1(a) but for the 13 bias frame set taken at -60°C.

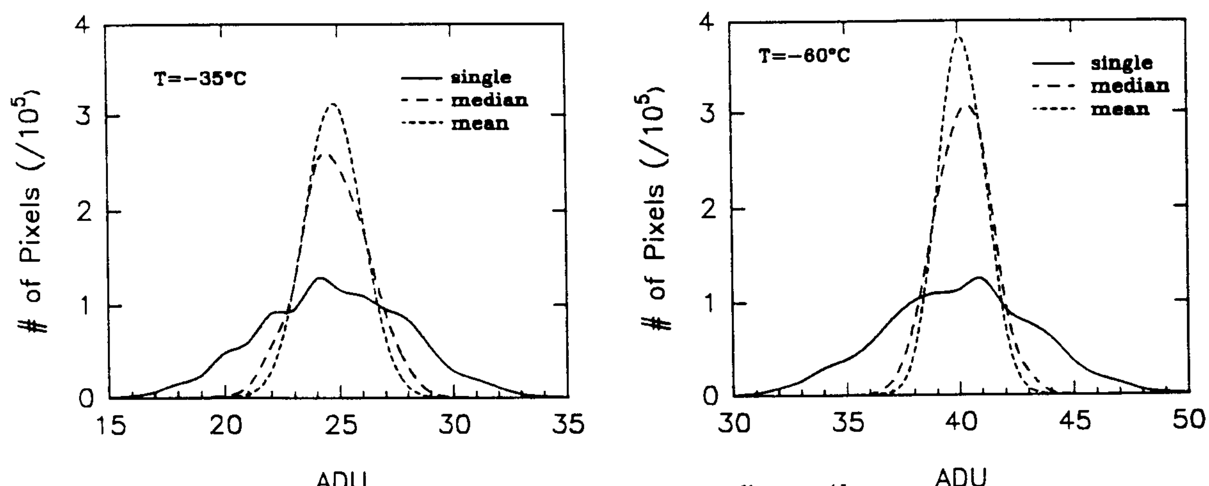


figure 1a figure 1b

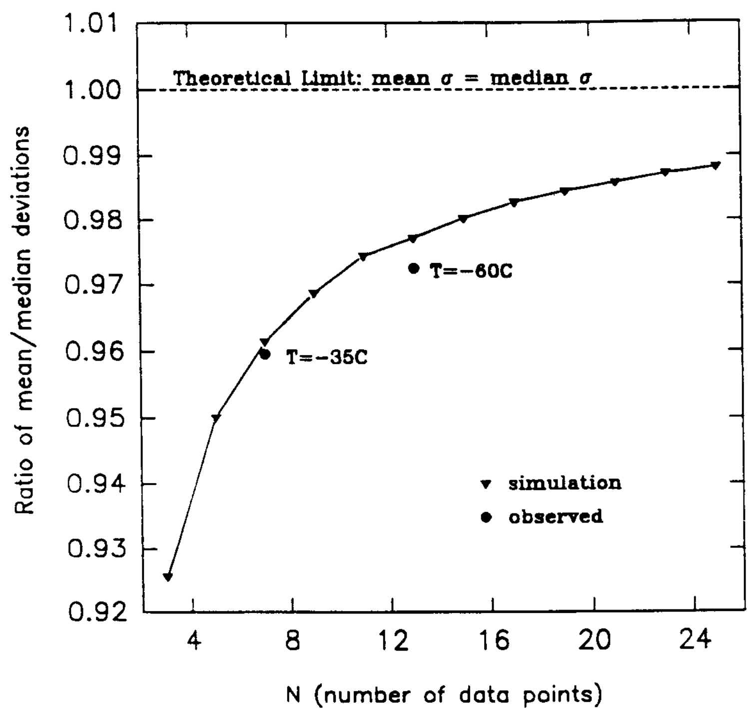
For each pixel the individual *mean* was calculated using data from the same pixel in the 7 or 13 frame set, and with this the individual sum of the squares of the residuals was calculated for each pixel, i.e.

$$y_{ij} = \sum_{k=1}^{N} (x_{ijk} - \bar{x}_{ij})^2$$

where N is the number (i.e. 7 or 13 here) of individual pixel data points (x_{ij}), and $\bar{x}_{ij}$ is the mean of the data set at each pixel location ij. Finally the average value for the sum of the squares of the residuals of all the pixels across the entire central region of the CCD was calculated. This procedure eliminated problems associated with the known spatially varying DC bias levels across the CCD. Such variation would contribute an extra term to the noise if the residuals were estimated by spatially averaging across the frame. This procedure was then used to find the *median* of the 7 and 13 frame sets, the sum of the squares of the residual about the median was calculated, and finally the average value of the residual calculated. The ratio of the average value of the sum of residuals squared using the mean to that using the median is shown in figure 2 for the -35°C and -60°C data sets. In both cases, the data indicates that the noise is well described by a gaussian form. An actual fit of a gaussian form to bias frame histograms, shown in figure 1, gives a standard deviation of $\approx 17\ e^-$, which is also the single frame rms read-out noise.

The theoretical estimation of the ratio of mean to median deviations for large N was obtained from equation (2) given above. In order to estimate this ratio for small values of N a simulation was used. A number N (N=3,5,7,9 .. 25) values were drawn from a random gaussian distribution, a mean and a median were calculated, and the sum of the squares of the residuals was obtained for each case. The procedure was repeated with a thousand such drawings at each N. An average value for the ratio was so obtained at each N. The simulation values and the exact expression given in equation (2) agree for large N. The solid line in figure 2 is the result from the simulation. As can be seen the ratios of the variance of the residuals for the experimental data (2 points) fall very close to the ratio derived from the noise simulation, for a gaussian distribution (the line).

Figure 2 : Ratio of the average sum of the residuals squares calculated using the mean to that using the median. Curve shows results of a simulation using number of data points indicated. The two marked points are for actual CCD data at the chip temperature indicated.



4. DARK CURRENT

In the presence of finite read noise and a low photon arrival rate from the scene of interest, it is often necessary to collect photons (integrate) for an extended period of time. While the signal photons are accumulating, charge caused by the finite temperature of the silicon lattice also accumulates. This extra signal is present with or without the presence of external photons and is hence referred to as "dark". The dark signal varies vastly between different pixels on the same CCD chip and is a strong function of temperature.

In order to investigate the dark current behavior of the Kodak CCD at reduced temperature, two sequences of exposures, with the shutter ahead of the Kodak CCD closed (i.e. the CCD is in the dark) were taken. The first sequence had the array cooled to -35°C at which time eight 600 second dark exposures were made. The second sequence had the array cooled to -60°C and consisted of two 600 second and two 1800 second exposures. In all cases a mean bias was removed from the exposure and either a median filter was applied to the sequence to remove cosmic ray events or such events were removed manually. Only the results from the central 1000 x 1000 pixels are used in what follows. The histogram of the resultant dark frame at -35°C is shown in figure 3. A fit of a gaussian form to the left hand edge of this observed histogram was made. This edge consists of those pixels with readout values less than zero after mean bias subtraction, thus implying they have little dark signal (which would drive the values positive), and are the result of only read-noise. A σ for the gaussian form equal to that found from fits to single bias frame divided by the square root of the number of independent frames ($\sqrt{8}$ in this case), was used. This gaussian form is also shown in figure 3.

Clearly a large amount of dark signal is seen above the noise floor imposed by the presence of read noise as measured from the bias frames. Figures 4(a) and 4(b) illustrate how much the dark current is reduced by cooling this chip to -60°C. Again a histogram of the expected distribution due to just read-out noise is shown for reference with its width determined by the single frame read noise and the number of frames used to generate the observed histogram. Figure 5 plots the histogram for two dark frames, one taken at -35°C and the other at -60°C, both with integration times of 600 seconds. The mean bias has been removed from both. The noise distribution is slightly different between the two curves since the -35°C data is the average of 8 frames while the -60°C data is the average of only 2 frames. The number of pixels with dark current greater than 0.085 e–/sec is ≈ 100,000 at -35°C and reduced to only ≈ 400 at -60°C.

Figure 3 : The histogram of the median of eight 600 second dark frames with the CCD held at -35°C. The dashed line is the expected gaussian distribution for the read-out noise as determined from bias frames.

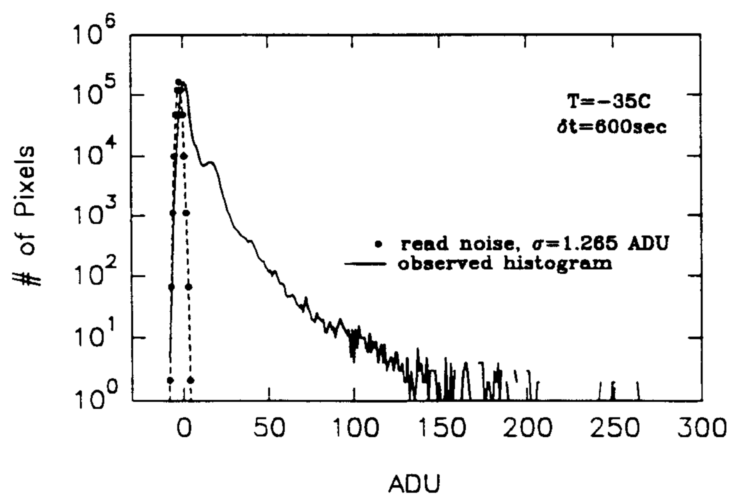


Figure 4 : The dark current obtained with the Kodak CCD at -60°C for integration times of 600 seconds and 1800 seconds. The expected distribution from just read noise is again shown.

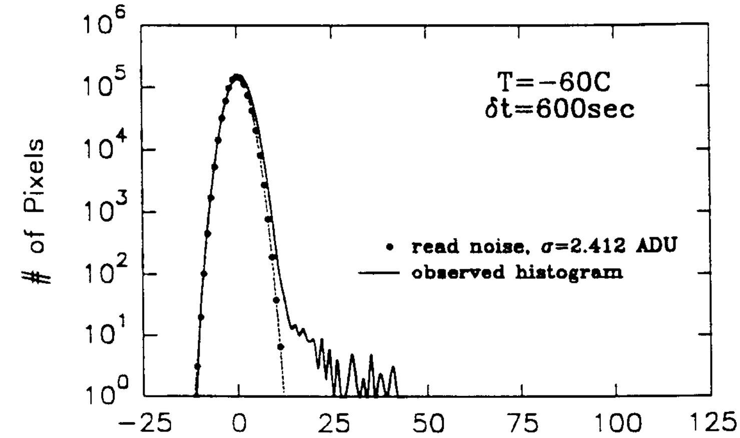


figure 4a

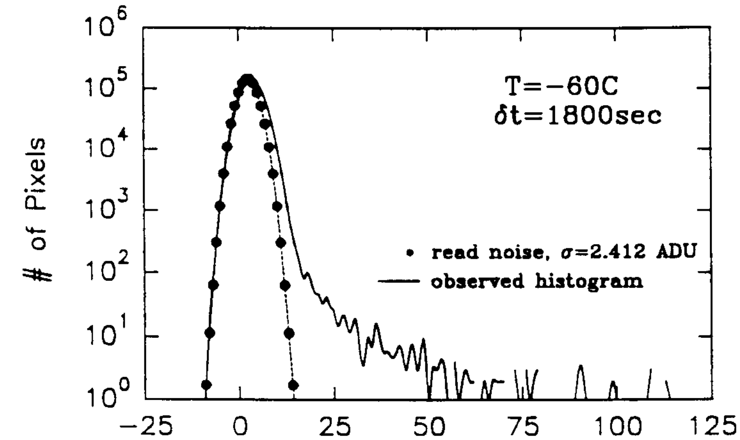
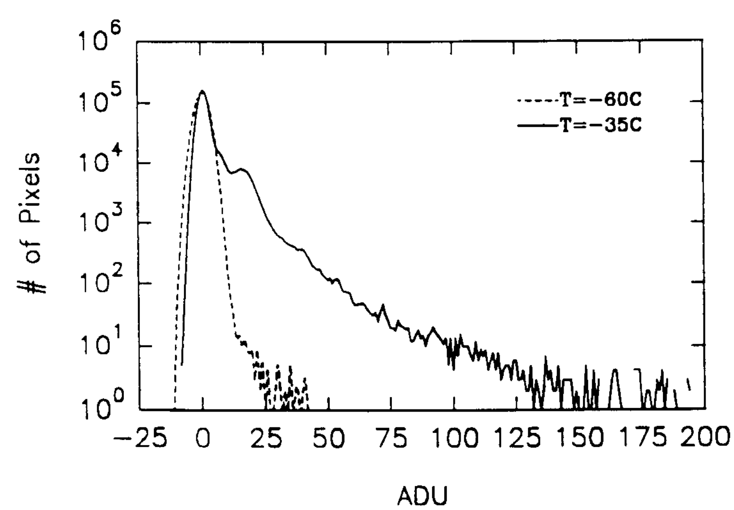


figure 4b

Figure 5 : A comparison of histograms of 600 second dark frames obtained with the CCD at -35°C and -60°C.



5. RESIDUAL IMAGES

Under very particular conditions the existence of residual images has been noticed with this CCD. Figure 6(a) shows the initial image (a star field) after it has had a mean bias removed. At this time the CCD was operated at -35°C and a filter that used 3 mm of RG830 Schott glass and 2 mm of WG305 (referred to as a Z filter), that transmits with an efficiency of > 80% at wavelengths longer than 850 nm, was used to restrict the incident radiation on the CCD. The exposure time for this image was 300 seconds and it resulted in the brightest object in the field being ≈ 5 times over saturated. The next image taken with the CCD after that shown in fig 6(a) was of a dark frame (shutter closed) of duration 300 seconds. The resultant dark is presented in fig 6(b) and residual images at the location of the brightest stars in figure 6(a) are clearly present. The difference in the central exposure times between the images shown in figure 6(a) and 6(b) is 10 minutes. On closer inspection the residual images look slightly more elongated in the direction across the page. In fact prior to the exposure shown in figure 6(a), a 60 second exposure of this same field had been obtained (the mid-exposure time was 6 minutes prior to the image shown in figure 6(a)). This earlier exposure was slightly spatially offset to the right of the image in figure 6(a). The elongation can then be attributed to a residual image from this prior exposure. The span of time (mid-exposure to mid-exposure) from the first 60 second exposure to the dark exposure is 16 minutes. The actual output counts, for the region in the vicinity of the brightest star that can be seen in figure 6(a), is given in table 2. Each of the numbers in the table 2,3 and 4 is a block average of 4 x 4 physical pixels. Table 3 gives the actual counts in the following dark exposure, and table 4 is the difference between this dark and a median true dark, that was free of any residual images. This residual image phenomena would affect using the CCD for photometry at the ≈ 0.1 % level. This number reflects the measured fraction of the original image charge remaining in the residual image.

This phenomena was further tested using a filter with a center wavelength of 430 nm and a FWHM of 80 nm (referred to as a B filter, consisting of 2mm of GG385 + 1mm of BG12 + 2mm of BG39). No clear residual image was seen under these circumstances. In addition the CCD was cooled to -60°C and the experiment with both the B and Z filters was attempted but no residual image was seen. The Eastman Kodak Company has operated and tested these devices at room temperature and found no evidence of image lag (as stated in their Technical Specifications for the KF-4200 chip)

The phenomena is probably best explained as attributable to bulk traps[7]. Long wavelength radiation that penetrates deep into the silicon is absorbed and the carriers generated can be trapped at problem locations in the lattice. At room temperature enough thermal energy is available so that the carriers can fall into the trap and jump out easily. At -35°C carriers fall into the trap but have only just enough energy to escape. It takes some time for all the trapped carriers to escape. At -60°C the carriers that fall into these traps never escape as their thermal energy is low. Short wavelength blue photons (including "B" filter photons) are strongly absorbed in a thin upper region of the silicon. The far red photons (including "Z" filter photons) are absorbed in a greater volume of silicon where more defects are available for trapping the carriers. This would explain why the residual image effect is much stronger with the Z filter than the B filter.

Table 2

Initial Illumination by a star field (raw AD Units). Each number in the table represents a 4x4 block average of physical pixels as is the case for tables 3 and 4.

107	130	175	239	286	311	284	216	166	126
131	185	272	405	510	591	531	368	241	164
173	286	479	762	1068	1752	1228	739	416	248
245	461	969	1849	3309	4095	3465	2004	816	406
374	844	2445	3935	4095	4095	4095	3916	1918	716
519	1760	3997	4095	4095	4095	4095	4095	3170	992
601	2428	4095	4095	4095	4095	4095	4082	2862	976
598	1829	3768	4095	4095	3968	3572	2529	1302	687
474	856	1385	1792	1842	1616	1249	887	620	422
276	370	462	567	635	598	548	445	325	236

Table 3

Dark Exposure Taken Immediately after Saturation Exposure (raw AD Units)

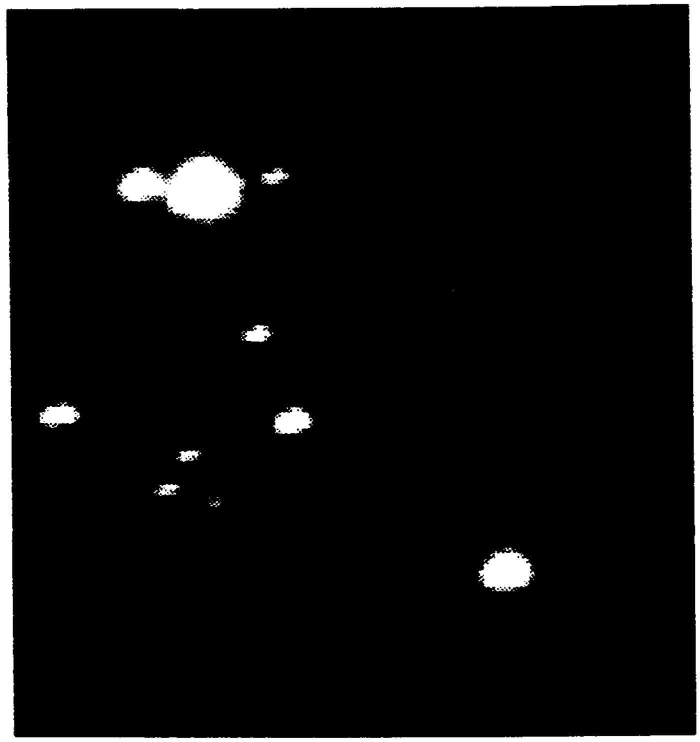
25	26	24	26	26	25	25	27	26	27
25	27	25	28	27	28	25	27	25	25
26	27	27	26	27	29	27	27	27	25
24	27	25	29	44	31	29	28	28	29
28	27	27	32	29	33	29	30	29	30
24	33	29	32	31	29	30	27	30	28
28	28	30	32	31	30	33	28	28	29
29	28	29	30	31	33	29	30	30	28
26	26	29	26	26	26	28	25	27	28
24	25	23	27	26	24	29	26	26	29

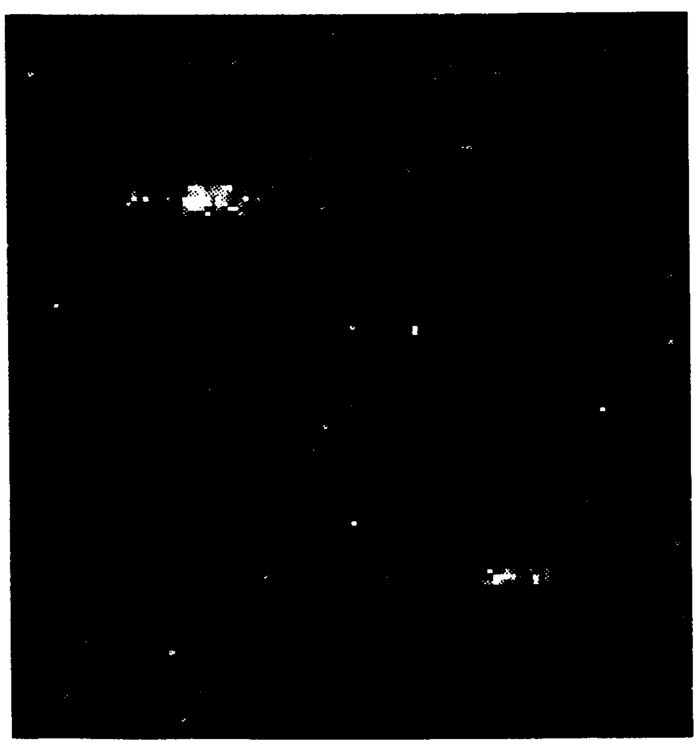
Table 4

Difference between a dark frame containing residual image and a mean true dark (AD Units)

0	0	-1	0	0	1	0	1	0	1
0	-1	0	0	0	0	0	0	0	-1
1	0	1	0	2	1	1	1	2	0
0	1	0	3	3	3	1	2	2	3
1	0	1	2	3	5	2	2	2	3
0	2	3	6	4	3	4	2	4	2
1	1	5	4	3	4	4	2	3	3
0	0	2	4	5	3	1	1	2	1
0	0	2	0	1	0	3	0	0	1
-1	0	-2	1	0	-1	1	0	0	0

Figure 6 : Upper Image is the initial 600 second image of a star field taken through a Z filter. Lower image is the following 600 second dark exposure (shutter closed) showing residual images at location of saturated stars seen in upper image. The images are the result of a 4x4 block average of 484 x 524 physical pixels.





<u>6. FLAT FIELD</u>

The response of a CCD is spatially variable across the chip. This is normally corrected by imaging a uniformly radiating source (e.g. a white reflector, or, the dawn or dusk sky, hence the name flat field) and using it as described in equation (1). In addition the response of the CCD can vary significantly as a function of the wavelength of the incident radiation. Images of a reflective barium sulfate screen that was illuminated by daylight corrected light bulbs were obtained through B and Z filters while the CCD was mounted at the cassegrain focus of a 24" f / 15 telescope. The two flat fields so obtained (a mean bias was subtracted from each) were divided to highlight spatial variations in the response difference. The histogram of this ratio is shown in figure 7 and a grayscale image of the ratio is shown in figure 8. The majority of pixels cluster around a single value for the ratio of the counts at the two wavelengths. The width of the distribution of this ratio is about 7% over the 10^6 central pixels. Over any smaller region this width would be significantly smaller. The same effect is seen with uniform fields gathered through filters with central passbands between the B and Z filters. For the highest accuracy it is therefore important to ensure that flat fields are taken using the same filter as used to gather the data frame.

<u>Figure 7</u> : Histogram of the central 1000 x 1000 pixels of the frame resulting from the division of the B flat field by the Z flat field.

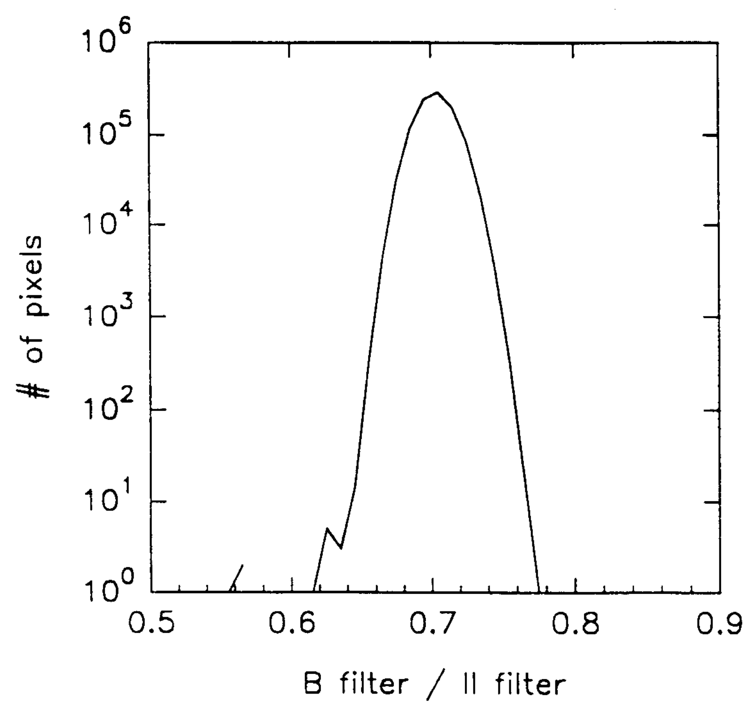
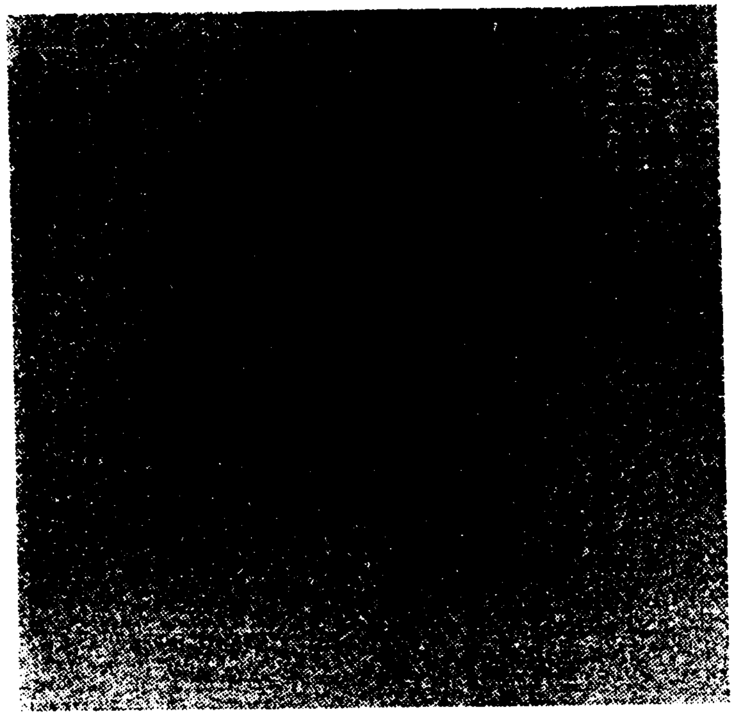


Figure 8 : Gray scale image of the central 1000 x 1000 portion of the resultant image after the division of a B flat field image by a Z flat field image. To highlight the variations the image has been block averaged using a 4 x 4 filter.



7. SUMMARY

A review of some practical considerations with respect to the data gathered using a large format CCD is presented. The identification of a residual image effect in these arrays is described and should be noted by those needing to make high precision photometric measurements.

8. ACKNOWLEDGMENTS

This work was made possible in part by the IUCRC program sponsored by the NSF, the New York State Science and Technology Foundation, and the associated Industrial Sponsors. The computer assistance of Chen Tang at RIT is gratefully acknowledged as are the helpful comments of Tom Lee at Eastman Kodak.

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