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ANALYSIS OF THE IMAGE SIGNAL MODULATION AND NOISE CHARACTERISTICS OF LASER PRINTERS

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Electronic imaging systems often employ several technologies to detect, process, and display image information. For an understanding of limitations present and opportunities available during design, a consistent systematic analysis of signal modulation and noise degradation is essential. Signal-to-noise ratio techniques are well suited to this task, which is simplified since signal transformations are often sequential <1>.

Our aim here is to describe the effect of the signal modulation and noise sources on the image signal. A laser printer receives the input image in digital form and converts this to an analog function of time, which is used to modulate the laser exposure intensity. The signal, until it is written, is a one-dimensional transformation (either electronic or optical) of the intended image. In general, the effect of the signal processing on the final image is not isotropic and depends on the specific writing configuration (e.g., pixel size) <2,3>. We can, however, include the equivalent descriptors (i.e., transfer functions and noise sources) in terms of the output image dimensions, given a particular writing scheme. Two types of noise sources are considered; those independent of the signal, and those that are a function of its mean value. This is in contrast with aliasing errors <2> since they will also be functions of the image spectrum (autocorrelation).

MODEL

Figure 1 shows a schematic representation of an imaging system including an output printer. After the sensor the image is sampled and digitized (quantized). We will assume that the quantization levels are equally spaced in exposure and that no image compression is involved. The digital signal is converted to analog form, which now includes quantization noise n_q. The analog signal f modulates (multiplies) the laser output.[1] The spread function associated with the modulator includes the image blurring that occurs in the fast scan direction due to interpolation between pixel values. We make the assumption that the laser exposure source can be represented by a continuous intensity profile function and that the exposure is sufficient that the modulation step can be described by the multiplication of the function by the pixel value. If, however, the laser emits a very low exposure, we would need to address the statistics of individual photon amplification and scattering events <4>. Since the modulated laser is scanned across the photosensitive recording material, the finite beam profile (width) introduces an effective spread function into the signal path.

Laser intensity noise can arise from many physical sources <5-7>, and each laser type exhibits unique noise characteristics. This often necessitates compensation of the exposure fluctuations and beam position errors by the modulating electronics <8,9>. In general, however, there will be uncompensated intensity noise, and we approximate this by a stochastic source. It will be assumed that the beam intensity varies as a function of time about the mean beam profile. We model this as the multiplication of the beam intensity by a random variable. This has the effect of varying the spread function associated with the scanned laser beam as the image is written. Two digital image restoration techniques for similar types of distortion have been described by Ward and Saleh <10>.

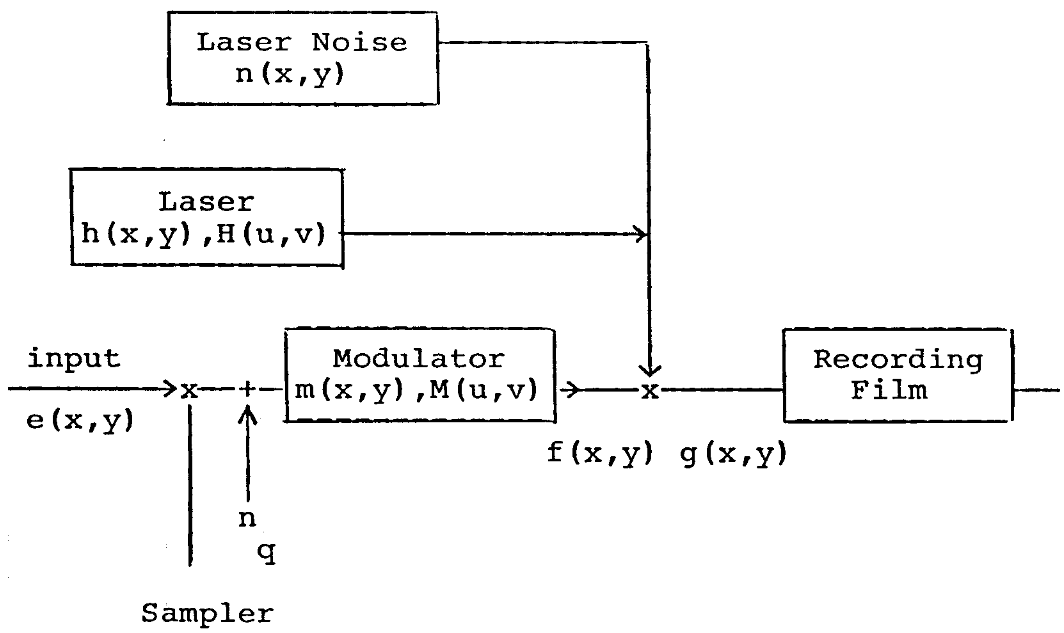

Figure 1. Schematic signal diagram for laser printer.

The modulated laser is then used to expose the recording material. In general, the choice of recording material will be influenced by system signal-to-noise ratio requirements rather than those for signal modulation transfer <11>.

ANALYSIS

The writer MTF, from detected input signal to output exposure incident on the recording material, can be expressed in terms of the MTF of each stage,

$$MTF(u,v) = M(u,v)\ H(u,v)$$

where M is the modulator MTF and H is the Fourier transform of the beam profile. We now analyze the noise due to sources identified in the imaging model. It is assumed that the laser printer input is a stationary random variable resulting from the detection of a uniform input exposure. This can be represented as the sum of a constant mean value plus a zero mean stochastic component,

$$e(x,y) = \mu_e + n_e(x,y)$$

where μ_e indicates the mean, or expectation, of e(x,y). Since we are not addressing aliasing noise due to either sampling or interpolation, the process of sampling, digitizing, and reconstructing of a continuous signal can be modelled as the addition of a quantization noise source. The noise due to quantization and rounding can be described <12,13> by a zero mean uniformly distributed random variable with variance,

$$\sigma_q^2 = XY\left[12\, 2^m\right]^{-1} \qquad \text{exposure}^2\ \text{mm}^2$$

where m is the number of bits used, and X and Y are the pixel sampling intervals in each image dimension. The output of the digital-to-analog converter is

$$f(x,y) = \left[\mu_e + n_e(x,y) + n_q(x,y)\right] * m(x,y)$$

where * indicates convolution and m is the modulator spread function. This quantization noise will be uncorrelated (white) for all signals except those that occupy very few levels, i.e., highly correlated signals. The corresponding noise power spectrum has a component due to each noise source,

$$S_f(u,v) = \left[S_e(u,v) + S_q(u,v)\right] |M(u,v)|^2. \qquad (1)$$

where S indicates the noise power spectrum and |.| the modulus. To understand the contribution of laser intensity fluctuations to image noise, we first express the modulated exposure in terms of the input signal and beam intensity profile. This can be represented as the convolution of the continuous signal with the beam profile function. This was implicit in equation (1). If the beam intensity fluctuates during image writing, this can be described as the multiplication of h(x,y) by a random variable, whose mean value is unity. Since the beam intensity fluctuations at a point affect the written pixel exposure at that position, the beam intensity fluctuations can be included in the convolution as follows,

$$g(x',y') = \iint_{-\infty}^{\infty} f(x,y)\; n(x,y)\; h(x-x',y-y')\; dxdy$$

where the mean of n(x,y) is unity. It can be shown that

$$\mu_g = \mu_f \; ,$$

$$\sigma_g^2 = \iiiint_{-\infty}^{\infty} R_f(x_1-x_2, y_1-y_2)\; R_n(x_1-x_2, y_1-y_2)\; h(x_1,y_1)$$

$$\cdot\, h(x_2,y_2)\; dx_1\, dx_2\, dy_1\, dy_2 \; + \; \mu_f^2 \iint_{-\infty}^{\infty} S_n(u,v)\,|H(u,v)|^2\, dudv$$

$$+ \; \mu_n^2 \iint_{-\infty}^{\infty} S_f(u,v)\,|H(u,v)|^2\, dudv$$

where R indicates the autocovariance function. The output noise power spectrum is given by

$$S_g(u,v) = (S_f(u,v) * S_n(u,v)) + \mu_f^2 S_n(u,v) + \mu_n^2 S_f(u,v) \; |H(u,v)|^2$$

or from equation (1),

$$S_g(u,v) = \Big[(\{S_e(u,v) + S_q(u,v)\}\,|M(u,v)|^2) * S_n(u,v) \; +$$

$$\mu_f^2 S_n(u,v) + \mu_n^2 \{S_e(u,v) + S_q(u,v)\}\,|M(u,v)|^2 \Big] \, |H(u,v)|^2 .$$

The final step in the imaging system is the laser exposure of the recording materials. System signal and noise transfer requirements are usually expressed in terms of the output image optical density or reflectance. This can be accomplished by assuming that the final printing step modifies the written exposure "signal" by a spread function and then adds signal-dependent granularity noise. This approach is due to Doerner <14>, who expressed the output Wiener spectrum as

$$WS(u,v) = \{\gamma\, 0.434\, |MTF_f(u,v)| \,/\, \mu_g\}^2 \; WS_g(u,v) + WS_f(u,v)$$

in units of Density^2 mm^2, where γ is the slope of the characteristic D-logE curve at exposure μ_g, and MTF_f and WS_f are the film MTF and Wiener spectrum, respectively.

CONCLUSIONS

An analysis has been given of image signal modulation and noise for a laser printer via a simple model that includes quantization, laser source intensity fluctuation, and interpolation. The effect of writer design choices can be expressed in a way that is consistent with established physical image quality measures. This can then be used to integrate the design of the hardcopy display step in the context of system image quality requirements.

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